HEA-003-1163005 Seat No. ________

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

November/December – 2017

Differential Geometry : EMT-3011

(New Course)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) Attempt all the questions.
(3) Figures to the right indicate full marks.

1 Attempt any seven : **14**

(1) Define : Regular curve.

(2) Define : Tangent vector field.

(3) Is the curve $\alpha(t) = \left(t^3, t^2, 2t\right)$ regular ? Justify your answer.

(4) Define : Arc length.

(5) Define : Unit speed curve.

(6) Define : The tangent space and the normal space.

(7) Define : Normal curvature and Geodesic curvature.

(8) Define : Simple surface.

(9) Define : Tangent vector to a simple surface.

(10) Define : Proper cp-ordinate patch.

2 Attempt the following : **14**

(a) Define : Reparametrization. If $g:[c,d]\rightarrow[a,b]$ is a reparametrization of a curve segment $\alpha:[a,b]\rightarrow R^3$ then show that the length of α is equal to the length of $\beta=\alpha\circ g$.

(b) Reparametrize the curve $\alpha(t)=(r\cos t, r\sin t, 0)$ by its arc length and also find its curvature (where $r>0$).

OR

(b) Find the arc length of the helix $\alpha(t)=(a\cos t, a\sin t, at\tan\alpha)$.

3 Attempt the following : **14**

(a) For the circular helix $\alpha(t)=(r\cos\omega s, r\sin\omega s, h\omega s)$, compute Frenet - Serret appartus (where $\omega=\left(r^2+h^2\right)^{-\frac{1}{2}}$)

OR

(a) Show that $\alpha(s)=\left(\frac{(1+s)^{\frac{3}{2}}}{3}, \frac{(1-s)^{\frac{3}{2}}}{3}, \frac{s}{\sqrt{2}}\right)$ is a unit speed curve and compute its Frenet - Serret appartus.

(b) Show that $\alpha(s)=\frac{1}{2}\left(\cos^{-1}s,\ s\sqrt{1-s^2},\ 1-s^2,\ 0\right)$ is a unit speed curve and compute its Frenet - Serret appartus.

4 Attempt the following : **14**

(a) State and prove Frenet - Serret theorem.

(b) Let $\alpha(s)$ be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k \neq 0$. Also if $\tau \neq 0$ then $\alpha - m = -\rho N - \rho' \sigma \beta$ and

$r^2 = \rho^2 + (\rho' \sigma)^2$ (where $\rho = \frac{1}{k}$ and $\sigma = \frac{1}{\tau}$).

5 Attempt any two : **14**

(a) If $x : u \to R^3$ is a simple surface and $f : v \to u$ is a co-ordinate transformation such that $y = x \circ f$ then prove that

(i) The tangent plane to the simple surface x at $P = x(f(a, b))$ is equal to the tangent plane to the simple surface y at $P = y(a, b)$.

(ii) The normal to the surface x at P is same as the normal to the surface y at P except possibly it may have the opposite sign.

(b) Prove that : The set of all tangent vectors to a simple surface $x : u \to R^3$ at P is a vector space.

(c) Find the co-efficient of second fundamental form and Christoffel symbols for the surface

$x(u', u^2) = (u', u^2, f(u', u^2))$.